

CS 412 Intro. to Data Mining

Chapter 3. Data Warehousing and On-line Analytical Processing

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Chapter 3: Data Warehousing and On-line Analytical Processing



Data Warehouse Modeling

OLAP Operations

Data Cube Computation: Concepts and Methods

Summary

What is a Data Warehouse?

- Defined in many different ways, but not rigorously
 - Support decision
 - Maintained Separately
 - Information processing
- "A data warehouse is a <u>subject-oriented</u>, <u>integrated</u>, <u>time-variant</u>, and <u>nonvolatile</u> collection of data in support of management's decision-making process."—W. H. Inmon
- Data warehousing:
 - □ The process of constructing and using data warehouses

Data Warehouse—Subject-Oriented

- Help make decisions
 - □ A **simple** and **concise** view (modeling and analysis)
 - Not details (transaction processing)
 - Organizing around major subjects, such as customer, product, sales
 - Excluding data that are not useful in the decision support process

Data Warehouse—Integrated

- Integrating multiple, heterogeneous sources
 - Ex. relational databases, flat files, on-line transaction records
- Consistency
 - Data cleaning and data integration techniques are applied.
 - Ex. Hotel price: differences on currency, tax, breakfast covered, and parking
 - U When data is moved to the warehouse, it is converted

Data Warehouse—Time Variant

Data Warehouse	Operational Database
Long time horizon (e.g., past 5-10 years)	current value data
Contains an element of time, explicitly or implicitly	data may or may not contain "time element"

Data Warehouse—Nonvolatile

- □ Independence A physically separate store
- Static No data management (updates, transaction processing, recovery, and concurrency control mechanisms)
- **Requires only two operations in data accessing:**
 - □ *initial loading of data* and *access of data*

OLTP vs. OLAP

- OLTP: Online transactional processing
 - DBMS operations
 - Query and transactional processing
- OLAP: Online analytical processing
 - Data warehouse operations
 - Drilling, slicing, dicing, etc.

	OLTP	OLAP
users	clerk, IT professional	knowledge worker
function	day to day operations	decision support
DB design	application-oriented	subject-oriented
data	current, up-to-date	historical,
	detailed, flat relational	summarized,
	isolated	multidimensional
		integrated, consolidated
usage	repetitive	ad-hoc
access	read/write	lots of scans
	index/hash on prim. key	
unit of work	short, simple	complex query
	transaction	
# records accessed	tens	millions
#users	thousands	hundreds
DB size	100MB-GB	100GB-TB
metric	transaction throughput	query throughput, response

Data Warehouse: A Multi-Tiered Architecture

- Top Tier: Front-End Tools
- Middle Tier: OLAP Server
- Bottom Tier: DataWarehouse Server

Data



Three Data Warehouse Models

Enterprise warehouse - Specially designed for the entire organization

Data Mart

- Specific set of subjects, selected groups of users
- Independent vs. dependent (directly from warehouse) data mart

Virtual warehouse

- A set of views over operational databases
- Only some of the possible summary views may be materialized



https://www.guru99.com/data-warehouse-vs-data-mart.html

Extraction, Transformation, and Loading (ETL)

Data extraction

get data from multiple, heterogeneous, and external sources

Data cleaning

detect errors in the data and rectify them when possible

Data transformation

convert data from legacy or host format to warehouse format

Load

sort, summarize, consolidate, compute views, check integrity, and build indicies and partitions

Refresh

propagate the updates from the data sources to the warehouse

Metadata Repository

- Meta data is data about data. It stores:
 - Description of structure (schema, etc.)
 - Operational meta-data
 - data lineage (history of migrated data and transformation path), currency of data (active, archived, or purged), monitoring information (warehouse usage statistics, error reports, audit trails)
 - The algorithms used for summarization
 - □ The mapping from operational environment to the data warehouse
 - Data related to system performance
 - warehouse schema, view and derived data definitions
 - Business data
 - business terms and definitions, ownership of data, charging policies

From Data Warehouse to Data Lake

- Data lake: a single repository of all enterprise data in the natural format
- Relational data, semi-structured data (e.g., XML, JSON), unstructured data (e.g., emails, Pdf files) and binary data (e.g., images, videos, audio)
- Data lake vs. data warehouse
 - Data Warehouse: top-down, structured and centralized
 - Data Lake: bottom-up, quick prototyping and democratic
- Data storage in data lake
 - Mandatory layers:
 - raw data, cleansed data, and application data
 - Optional layers
 - standardized data layer & sandbox data layer



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Summary

From Tables and Spreadsheets to Data Cubes

- A data warehouse is based on a multidimensional data model which views data in the form of a data cube
- Main function is to provide summarizations of the data
 - E.g., summarize the units or dollars sold at a particular store over a particular time period
- Can compute summarizations online (as they are requested)
 - Can be very slow
- Better to pre-calculate some summarizations

Design of Data Warehouses

- Dimension tables, such as item (item_name, brand, type), or time(day, week, month, quarter, year)
- Fact table contains measures (such as dollars_sold) and keys to each of the related dimension tables
- Different schema exist
 - Star
 - Snowflake
 - Fact constellation



Conceptual Modeling of Data Warehouses

- Modeling data warehouses: dimensions & measures
 - □ <u>Star schema</u>
 - Snowflake schema
 - Fact constellations

Star Schema: An Example



Snowflake Schema: An Example

A refinement of star schema where some dimensional hierarchy is normalized into a set of smaller dimension tables, forming a shape similar to snowflake



Fact Constellation: An Example



Data Cube: A Lattice of Cuboids



Data Cube: A Lattice of Cuboids



Data Cube: A Lattice of Cuboids



Calculating Number of Cuboids

- Consider dimensions as binary numbers
- Example: 4 dimensions
 - **Each** is either in the cuboid, or not in the cuboid
 - \Box (, , ,) \leftarrow choice of 0 or 1 for each element of vector
 - **Sum up for each position:** $2^3 + 2^2 + 2^1 + 2^0 + 1$ (0-d cuboid) = 2^4
- In general, 2^d cuboids (d = number of dimensions)

A Concept Hierarchy for a Dimension (location)



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Summary

Multidimensional Data

□ Sales volume as a function of product, month, and region

Dimensions: *Product, Location, Time* **Hierarchical summarization paths**





A Sample Data Cube





Cuboids Corresponding to the Cube



How can we play with the Cube?

Roll up & Drill down



Dice and Slice



Other Typical OLAP Operations

Pivot (rotate):

reorient the cube, visualization, 3D to series of 2D planes

Drill across:

involving (across) more than one fact table

Drill through:

through the bottom level of the cube to its back-end relational tables (using SQL)

Data Cube Measures: Three Categories

- Distributive: if the result derived by applying the function to n aggregate values is the same as that derived by applying the function on all the data without partitioning
 - E.g., count(), sum(), min(), max()
- Algebraic: if it can be computed by an algebraic function with M arguments (where M is a bounded integer), each of which is obtained by applying a distributive aggregate function
 - avg(x) = sum(x) / count(x)
 - How about standard_deviation()
- $s^{2} = \frac{1}{n-1} \sum_{i=1}^{n} (x_{i} \bar{x})^{2} = \frac{1}{n-1} \left[\sum_{i=1}^{n} x_{i}^{2} \frac{1}{n} (\sum_{i=1}^{n} x_{i})^{2} \right]$
- Holistic: if there is no constant bound on the storage size needed to describe a subaggregate.
 - E.g., median(), mode(), rank()

Efficient Data Cube Computation

- If I have *n* dimensions, each with L_i levels, how many cuboids are needed to preprocess all?
- Calculating **all** cuboids is costly in computation and time.
- How to decide which cuboid be pre-calculated (Materialization)?
 - Based on size of data, sharing, access frequency, etc.
 - Example: I know my users always search by Quarter, so that cuboid should be pre-calculated.
 - Example: If I pre-calculate days, I can use days as input to Months (30 or 31 days), or weeks (7 days), etc.





Indexing OLAP Data

Indexing

- Main purpose of indexing is to make the calculation faster/efficient
- **Common Warehouse Index:** <u>Bitmap Index</u>
 - Benefits in Warehousing:
 - Reduced response time for large classes of ad hoc queries.
 - Reduced storage requirements compared to other indexing techniques.
 - Dramatic performance gains even on hardware with a relatively small number of CPUs or a small amount of memory.

https://docs.oracle.com/database/121/DWHSG/schemas.htm#DWHSG9041
Indexing OLAP Data: Bitmap Index

- Index on a particular column
 - Each value in the column has a bit vector: bit-op is fast
 - The length of the bit vector: # of records in the base table
 - The *i*-th bit is set if the *i*-th row of the base table has the value for the indexed column
 - Not suitable for high cardinality domains. (WHY?)
- A recent bit compression technique, Word-Aligned Hybrid (WAH), makes it work for high cardinality domain as well [Wu, et al. TODS'06]

Dase table				
Cust	Region	Туре		
C1	Asia	Retail		
C2	Europe	Dealer		
C3	Asia	Dealer		
C4	America	Retail		
C5	Europe	Dealer		

Baca tabla



Index on TypeRecIDRetailDealer110201301410501

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Data Cube: A Lattice of Cuboids



40

Data Cube: A Lattice of Cuboids



Base vs. aggregate cells Ancestor vs. descendant cells Parent vs. child cells □ (*,*,*,*) ∕ 0-D (agg) □ (*, milk, *, *) 1-D (agg) Ъ (*, milk, Urbana, *) 2-D (agg) (*, milk, Chicago, *) 2-D (agg) □ (9/15, milk, Urbana, *) 3-D (agg) (9/15, milk, Urbana, Dairy land) 4-D (base)

Cube Materialization: Full Cube vs. Iceberg Cube

□ Full cube vs. iceberg cube

compute cube sales_iceberg as
SELECT month, city, customer_group, COUNT(*)
FROM salesInfo
CUBE BY month, city, customer_group
HAVING count(*) >= min support





- Compute only the cells whose measure satisfies the iceberg condition
 - **Ex.:** Show only those cells whose count is at least 100
- Only a small portion of cells may be "above the water" in a sparse cube

Why Iceberg Cube?

- No need to save nor show those cells whose value is below the threshold (iceberg condition)
- Efficient methods may even avoid computing the un-needed, intermediate cells
- Avoid explosive growth

Example

Example: A cube with 100 dimensions

- □ Suppose it contains only 2 base cells: {(a₁, a₂, a₃, ..., a₁₀₀), (a₁, a₂, b₃, ..., b₁₀₀)}
- □ How many aggregate cells if "having count >= 1"?
 - □ Answer: (2¹⁰¹ 2) 4 (Why?!)

Example

Example: A cube with 100 dimensions

- □ Suppose it contains only 2 base cells: {(a₁, a₂, a₃, ..., a₁₀₀), (a₁, a₂, b₃, ..., b₁₀₀)}
- What about the iceberg cells, (i,e., with condition: "having count >= 2")?
 Answer: 4 (Why?!)

Is Iceberg Cube Good Enough? Closed Cube & Cube Shell

- Let cube P have only 2 base cells: $\{(a_1, a_2, a_3 \dots, a_{100}): 10, (a_1, a_2, b_3, \dots, b_{100}): 10\}$
 - □ How many cells will the iceberg cube contain if "having count(*) \ge 10"?
 - □ Answer: $2^{101} 4$ (still too big!)
- **Closed cube:**
 - A cell c is *closed* if there exists no cell d, such that d is a descendant of c, and d has the same measure value as c
 - Ex. The same cube P has only 3 closed cells:
 - $\Box \{(a_1, a_2, *, ..., *): 20, (a_1, a_2, a_3 ..., a_{100}): 10, (a_1, a_2, b_3, ..., b_{100}): 10\}$
 - □ A *closed cube* is a cube consisting of only closed cells
- **Cube Shell:** The cuboids involving only a small # of dimensions, e.g., 2
 - Idea: Only compute cube shells, other dimension combinations can be computed on the fly

Roadmap for Efficient Computation

- General computation heuristics ^[1]
- Computing full/iceberg cubes: 3 methodologies
 - Bottom-Up:
 - □ Multi-Way array aggregation ^[2]
 - Top-down:
 - □ BUC ^[3]
- High-dimensional OLAP:
 - □ A Shell-Fragment Approach ^[4]
- Computing alternative kinds of cubes:
 - Partial cube, closed cube, approximate cube,

- 1. (Agarwal et al.'96)
- 2. (Zhao, Deshpande & Naughton, SIGMOD'97)
- 3. (Beyer & Ramarkrishnan, SIGMOD'99)
- 4. (Li, et al. VLDB'04)

Efficient Data Cube Computation: General Heuristics

- Sorting, hashing, and grouping operations are applied
 - **Share-sorts**
 - **Share-partitions**

Reuse

- Smallest-child: computing a cuboid from the smallest, previously computed cuboid
- Cache-results: caching results of a cuboid from which other cuboids are computed to reduce disk I/Os
- Amortize-scans: computing as many as possible cuboids at the same time to amortize disk reads



S. Agarwal, R. Agrawal, P. M. Deshpande, A. Gupta, J. F. Naughton, R. Ramakrishnan, S. Sarawagi. On the computation of multidimensional aggregates. VLDB'96

Multi-Way Array Aggregation (MOLAP)

- How can I efficiently calculate all group-by cell aggregations? Full cube computation
- Fundamental Concept: AB, AC, and BC can be computed from ABC. A, B, and C can be computed from AB/AC/BC.
- Common Practice with limited memory: Do not load the entire dimension (in multi-way array form) into memory at once. Use Chunks:



http://pages.cs.wisc.edu/~nil/764/DADS/38_zhao97array based.pdf - Zhao et al. '97



Multi-Way Array Aggregation (MOLAP)

- Chunk is stored as(chunk_id, offset)
 - Tells which cells in the chunk have data
- Goal: Read chunk only once in memory
 - BC /AB only once
- **Example:** Student Record Data Warehouse
- count(A) > count (B) > count(C)
- What is best order to put the chunks in order to calculate the aggregation?

Example:
A: 4000, B: 400, C: 40
Chunk:
1000 x 100 x 10



- □ Scan Order: 1 2 3 4 5 6 ...
- Goal: Fully compute chunk only once



- □ While we scan through 1..4
 - One row of AC plane is partially computed
 - One chunk of BC plane is fully computed (write to file)
 - One row in AB plane is partially computed
- now scan through 5...8

- □ Scan Order: 1 2 3 4 5 6 ...
- Goal: Fully compute chunk only once



- □ While we scan through 5...8
 - Same **row** of AC plane is updated
 - Another chunk of BC plane is fully computed (reuse the same place in memory)
 - another row in AB plane is partially computed
- Continue on 9...12
- Continue on 13...16

- □ Scan Order: 1 2 3 4 5 6 ...
- Goal: Fully compute chunk only once



- □ While we scan through 13...16
 - One row of AC plane is fully computed (write to file)
 - Another chunk of BC plane is fully computed (reuse the same place in memory)
 - Whole AB plane is partially computed
- Memory requirement:
 - 4000 x 10 (AC) + 100 x 10 (BC) +
 4000 x 400 (AB) = 1,641,000 units

□ Dimension Order: 1 – 5 – 9 – 13 – 2 – 6 – ...



- One column of BC plane is fully computed (write to file)
- Another chunk of AC plane is fully computed (reuse the same place in memory)
- Whole AB plane is partially computed
- Memory:
 - 400 x 10 (BC) + 1000 x 10 (AC) + 4000 x 400 (AB)
 - **1,614,000** units



- One row of AC plane
- One chunk of AB plane
- All chunks in BC plane
- Memory:
 - 1000 x 40 (AC) + 1000 x 100 (AB) + 400 x 40 (BC)
 - □ 156,000 units
 - The best order

- Main Goal of Multi-Way: Reducing memory and I/O
 - How?
 - □ Keep the smallest plane in main memory
 - Fetch and compute only one chunk at a time for the largest plane
 - The planes should be sorted and computed according to their size in ascending order
 - □ Suppose A>B>C>...

for a in A: for b in B: for c in C: ...

- Pros and Cons of Multi-Way
 - **Pro:** Efficient for computing the **full cube** for a small number of dimensions
 - **Con:** Can not calculate iceberg cube.
 - i.e: If there are a large number of dimensions, "top-down" computation and iceberg cube computation methods (e.g., BUC) should be used

Cube Computation: Computing in Reverse Order

- Iceberg cube computation
- **BUC (Beyer & Ramakrishnan, SIGMOD'99)**
 - Bottom-Up (cube) Computation
 - "top-down" in our view since we put Apex cuboid on the top!
- Divides dimensions into partitions and facilitates iceberg pruning
 - Prune if not satisfy min_sup
 - □ If *minsup* = 1, compute full CUBE!
- No simultaneous aggregation



BUC: Partitioning and Aggregating

- Cannot fit in main memory
 - Sort *distinct* values and partition to fit
 - Aggregation when sorting
 - Continue processing
- Iceberg cube
 - □ If count of (a1, b1, *, *, *) < min_support
 - No need to sort on C



MultiWay VS BUC

	multiway	BUC
Input format	Multi-dimensional array	Relational database
Good for	Full cube	Iceberg cube
Key idea	Simultaneously Aggregation	Partition and sort
Calculation direction	A B C BC BC ABC ACC BC	1 all 2 A 10 B 14 C 16 D 3 AB 7 AC 9 AD 11 BC 13 BD 15 CD 4 ABC 6 ABD 8 ACD 12 BCD 5 ABCD

High-Dimensional OLAP?—The Curse of Dimensionality

- □ High-D OLAP Applications:
 - **E**.g. bio-data analysis, statistical surveys
- None of the previous cubing method can handle high dimensionality!
 - Iceberg cube and compressed cubes: only delay the inevitable explosion
 - Full materialization: still significant overhead in accessing results on disk
- A shell-fragment approach: X. Li, J. Han, and H. Gonzalez, High-Dimensional OLAP: A Minimal Cubing Approach, VLDB'04



A curse of dimensionality: A database of 600k tuples. Each dimension has cardinality of 100 and *zipf* of 2.

Fast High-D OLAP with Minimal Cubing

- Observation: OLAP occurs only on a small subset of dimensions at a time
- Semi-Online Computational Model
 - Partition the set of dimensions into shell fragments
 - Compute data cubes for each shell fragment while retaining **inverted indices**
 - Given the pre-computed fragment cubes, dynamically compute cube cells of the high-dimensional data cube online
- Major idea: Tradeoff between the amount of pre-computation and the speed of online computation
 - Reducing computing high-dimensional cube into precomputing a set of lower dimensional cubes
 - Online re-construction of original high-dimensional space
 - Lossless reduction

Computing a 5-D Cube with 2-Shell Fragments

Example: Let the cube aggregation function be count

TID	Α	В	С	D	Ε
1	a1	b1	c1	d1	e1
2	a1	b2	c1	d2	e1
3	a1	b2	c1	d1	e2
4	a2	b1	c1	d1	e2
5	a2	b1	c1	d1	e3

- Divide the 5-D table into 2 shell fragments:
 - □ (A, B, C) and (D, E)
- Build traditional invert index (1-D)

Attribute Value	TID List	List Size
al	123	3
a2	4 5	2
b1	145	3
b2	2 3	2
c1	12345	5
d1	1345	4
d2	2	1
e1	12	2
e2	34	2
e3	5	1

Shell Fragment Cubes: Ideas

Shell-fragment

a1

a1

a2

a2

- Generalize the 1-D inverted indices to multidimensional ones in the data cube sense
- Compute all cuboids for data cubes ABC and DE while retaining the inverted indices
 - Ex. shell fragment cube ABC contains 7 cuboids:
 - □ A, B, C; AB, AC, BC; ABC
- This completes the offline computation
- ID_Measure Table
 - If measures other than count are present, store in *ID_measure* table separate from the shell fragments

	tid	count	sum
	1	5	70
	2	3	10
2	3	8	20
	4	5	40
	5	2	30

	Attribute Value		TID List	List Size	
	a1		23	3	
	a2	4 5	5	2	
	b1		15	3	
	b2	23	3	2	
	c1	12	2345	5	
	d1	13	3 4 5	4	
	d2	2		1	
	e1	12	2	2	
AB	e2	3 4	1	2	
	e3	5		1	
ell	Intersectio	on	TID List	List S	Size
b1	$123 \cap 14$	5	1	1	
b2	123 023		23	2	
b1	45∩14	5	4 5	2	
b2	45∩23		φ	0	

Shell Fragment Cubes: Size and Design

- Given a database of T tuples, D dimensions, and F shell fragment size, the fragment cubes' space requirement is: $O\left(T\left[\frac{D}{F}\right](2^F-1)\right)$
 - □ For F < 5, the growth is sub-linear
- Fragment groupings can be arbitrary to allow for maximum online performance
 - Known common combinations (e.g.,<city, state>) should be grouped together
- Shell fragment sizes can be adjusted for optimal balance between offline and online computation
- Shell fragments do not have to be disjoint

Attribute Value	TID List	List Size
a1	123	3
a2	4 5	2
b1	145	3
b2	2 3	2
c1	12345	5
d1	1345	4
d2	2	1
e1	12	2
e2	3 4	2
e3	5	1

Cell	Intersection	TID List	List Size
a1 b1	123 \cap 145	1	1
a1 b2	123 023	23	2
a2 b1	45∩145	4 5	2
a2 b2	45∩23	φ	0

Online Query Computation with Shell-Fragments

- A query has the general form: $\langle a_1, a_2, ..., a_n : M \rangle$
- Each a_i has 3 possible values
 - Instantiated value— this is what we want to look at
 - □ Inquire ? Function want to analyze these dimensions
 - □ Aggregate * function don't care about these dimensions
 - Ex: Suppose we want to query student data for junior (year 3) students and want to compare scores for different genders and ages, but don't care about what high school they attended.

<3, ?, ?, *: count>

Online Query Computation with Shell-Fragments

- Method: Given the materialized fragment cubes, process a query as follows
 - Divide the query into fragments, same as the shell-fragment
 - □ Fetch the corresponding TID list for each fragment from the fragment cube
 - Intersect the TID lists from each fragment to construct **instantiated base table**
 - Compute the data cube using the base table with any cubing algorithm



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Summary

- Data warehousing: A multi-dimensional model of a data warehouse
- Data Warehouse Modeling
 - Data Cube: a multidimensional data model
 - Star schema, snowflake schema, fact constellations
- OLAP operations: drilling, rolling, slicing, dicing and pivoting
- Data Cube Computation:
 - Basic Concepts: cuboids; iceberg cube; closed cube and cube shell, OLAP servers
 - Computation Methods: MultiWay Array Aggregation, BUC, High-Dimensional
 OLAP with Shell-Fragments

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72

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